

# GEOMETRIZATION OF THE LANGUAGE OF CHEMISTRY: APPROXIMATE SPATIAL MATHEMATICAL MODEL OF DIFFERENT SPECIES OF ATOMS AND MENDELEEV'S PERIODIC LAW

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## ABSTRACT

The foundations, development, and analysis of an approximate spatial mathematical model of different species of atoms and Mendeleev's law of periodicity are presented. The three- and two-dimensional mathematical images of basic chemical notions such as chemical element, group and period in the Periodic System, and others, are determined. The statics of the different species of atoms is axiomatized by formulating a set of definitions, axioms, theorem, lemma, corollaries and properties. A basic mathematical formalism for geometrization of the statics of simple chemical objects and a method for construction of spatial mathematical models of some basic chemical notions and relations are proposed and demonstrated for the approximate model.

## INTRODUCTION

In the first paper [1] of this series a system of three starting hypotheses, concerning the different evolutionary stages in the development of scientific branches and sciences is formulated. Using these hypotheses the particular problem for geometrization of the language of chemistry is formulated and grounded in a general form. The way for solving this problem is outlined by formulating a system of four basic problems to be solved during the next evolutionary stage of the development of the language of chemistry. Several aspects of these system of problems are briefly discussed in order to clarify their meaning and to introduce the main context needed. The main results, presented in detail in the monograph [2], are briefly reviewed.

In the second paper [3] it is shown that the fundamentals of the language of chemistry can be logically divided into four relatively separated parts everyone of which introduce the corresponding group of basic chemical notions. The central notions of the four parts and other basic notions are defined. The system of five basic assumptions playing the role of axiomatics in chemistry is formulated. The conceptual schemes, used for geometrization of each part of the fundamentals of chemistry, are formulated. Spatial mathematical models of the sets of different species of atoms and different species of monoatomic ions are briefly presented. As a result: (i) the problem of the geometrization of the language of chemistry is solved on a conceptual level; (ii) it is shown that it is possible to translate the structure of chemical notions and relations in entirely mathematical language.

The main aim of this paper is to propose one axiomatic formulation (i.e. to formulate a logical structure of definitions, axioms, theorem, lemma, corollaries, properties, etc.) of that part of the language of chemistry,

which introduces the set **S** of different species of atoms [2, 3]. To achieve this goal we introduce, on the example of an approximate spatial mathematical model, the basic mathematical formalism for geometrization of the statics of simple chemical objects and demonstrate the method for constructing spatial mathematical models of some basic chemical notions and relations. We found, construct and analyze one approximate spatial mathematical model of different species of atoms originating from a simplified tabular Short Form of Periodic System (SFPS). It should be pointed out that the approximate mathematical model and the starting simplified tabular SFPS must not be regarded beyond the purposes mentioned above. So, the approximate model is an intermediate working model towards more accurate mathematical models originating from “correct” versions of the Periodic System [2, 3]. These more accurate models present more precisely the proposed axiomatic formulation.

### FOUNDATIONS OF THE APPROXIMATE MODEL

In [3] we have pointed out that, from a logical point of view: (i) the identification of the notion SPECIES<sup>1</sup> OF ATOMS with that of CHEMICAL ELEMENT is a typical example of sense anachronism in the language of contemporary chemistry; (ii) the set **S** of DIFFERENT SPECIES OF ATOMS is that introduced first in chemistry; (iii) this set is introduced by the implicit definition of any three-dimensional form of the Periodic System (PS). On the other hand, it is well known that there exist a lot of different types of the Periodic System presenting the basic chemical law (the Mendeleev’s law) with different degree of accuracy and completeness [4]. Thus, each spatial mathematical model of chemical notions and relations should always be underlain by some previously chosen particular forms of PS. This particular form of PS is named **starting table** of the given mathematical model. We use one and the same method to construct the different mathematical models. In order this method to be applicable, the starting table should obey the following conditions: (1) A single row of symbols of chemical elements should correspond to each row of cells in the table; (2) A single column of symbols of chemical elements should correspond to each column of cells in the table.

All popular tabular forms of PS (short and long) satisfy the first condition. However, no tabular Short Forms of PS (SFPS) meets the second condition. At least two columns of symbols of chemical elements correspond to each column of cells in these tables. For this reason the known tabular SFPS could not be used as starting tables in the method proposed. On the other hand, it is important to construct a spatial mathematical model of the basic chemical notions based on a SFPS. More important reasons are:

- a) Almost all the symbols of chemical elements of maximal stoichiometric valence are placed in one and the same column of cells in the tabular SFPS, whereas in the long tables they are placed in two different columns of cells, separated from each other. In other words, one of the most widely used chemical notions, the “maximal stoichiometric valence”, is defined much better in the tabular SFPS.
- b) From a logical point of view, the most important reason to construct firstly this intermediate **approximate** mathematical model of the basic chemical notions (i.e. a model based on a **simplified** tabular SFPS) is the complexity of the logical analysis of the existing system of chemical notions.

We have already noted above that none of the known SFPS obeys the necessary conditions. That is why we constructed a suitable **simplified** tabular SFPS (see Fig. 1). In order to make it compact we have put

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<sup>1</sup> SPECIES in this study means **the major subdivision of a genus or subgenus, regarded as the basic category of chemical classification.**

lanthanides and actinides out of the table in separate columns. However, these columns should be considered as parts of the third column of the table.

The simplified tabular SFPS obeys the following conditions: (1) the order of the chemical elements, groups, and periods is similar to that in the popular eight-column tabular SFPS; (2) there is only one symbol of chemical element in each cell of the table; (3) the symbols of all chemical elements occupy equivalent positions in the corresponding cells of the table; (4) the symbols of chemical elements, belonging to one and the same group, belong to one and the same column of the table.

This table defines in a unique graphical way the closely related chemical notions “group of PS” and “maximal stoichiometric valence”. It should be noted that this condition is not strictly obeyed by any of the popular tabular SFPS, while the LFPS do not obey it at all.

It is easy to check that all most general regularities of the Periodic law, manifested in the popular eight-

PERIODS	GROUPS (COLUMNS)										
	I	II	III	IV	V	VI	VII	VIII			
1	1 H							2 He	1		
2	3 Li	4 Be	5 B	6 C	7 N	8 O	9 F	10 Ne	2		
3	11 Na	12 Mg	13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	3		
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	4	R	
								27 Co	5		
								28 Ni	6		
5	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	7	O	
	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	8		
								45 Rh	9		
6								46 Pd	10	W	
	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	11		
	55 Cs	56 Ba	57 La						12		
6			lan- tha- ni- des								↓
			71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	26	S	
								77 Ir	27		
								78 Pt	28		
7	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn	29		
	87 Fr	88 Ra	89 Ac						30		↓
			ac- ti- ni- des								
		103 Lr	104 Ku	105 Ns					44		

Lanthanides	Actinides
III 57 La	III 89 Ac
58 Ce	90 Th
59 Pr	91 Pa
60 Nd	92 U
61 Pm	93 Np
62 Sm	94 Pu
63 Eu	95 Am
64 Gd	96 Cm
65 Tb	97 Bk
66 Dy	98 Cf
67 Ho	99 Es
68 Er	100 Fm
69 Tm	101 Md
70 Yb	102 No
71 Lu	103 Lr

**FIGURE 1. Simplified tabular short form of the Periodic System (starting table T<sub>0</sub>)**

group SFPS, are manifested as well in the proposed simplified SFPS. The only important difference is that the latter does not account for the belonging of atoms to the corresponding **a** or **b** subgroup. This difference comes from the fact that the simplified tabular SFPS obeys conditions 2-4, whereas the popular tabular SFPS does not. On the other hand, this table is not designed to replace for practical purposes the well known table forms of PS, rather to introduce the basic mathematical formalism, as we already mentioned in the Introduction.

### CONSTRUCTION OF THE APPROXIMATE MODEL

We have already obtained one simplified tabular SFPS which allows us to construct a spatial mathematical model of the most general regularities of Mendeleev's periodic law, i.e. to prove:

**THEOREM 1.** There exists such a unique spatial mathematical model of the different species of atoms which, at the same time, is a veracious mathematical model of the simplified tabular SFPS.

This theorem plays the role of a **basic statement** for the approximate model and is directly proved by constructing the desired model. The proof of the theorem includes the following steps:

- I. Construction of an appropriate mathematical space that allows to construct the desired spatial mathematical model.
- II. Choice of an appropriate coordinate system (CS) in this space.
- III. Construction of the desired spatial mathematical model with respect to the previously chosen CS in the previously constructed space.
- IV. Check of the uniqueness and the veracity of the constructed mathematical model.

Now we shall prove the theorem. Let  $V_M(3)$  be a three-dimensional metric Euclidean space onto the field of the real numbers, in which each dimension has a different qualitative meaning (this meaning is clarified below). Further, let  $K_M=(O;z,\rho,\varphi)$  be a specially chosen cylindrical coordinate system in  $V_M(3)$ , satisfying the conditions that the coordinates  $z$ ,  $\rho$  and  $\varphi$  represent respectively the number of neutrons and protons in the simple chemical objects, and the periodicity of the change in the chemical properties of different species of atoms. (This complex periodicity is presented with different degree of accuracy and completeness in different forms of PS.) The origin  $O$  of the coordinate system  $K_M$  coincides with the **natural zero** point of  $V_M(3)$ .

The space  $V_M(3)$  defined above and the coordinate system  $K_M$  are named **Mendeleev's space** and **Mendeleev's coordinate system**. They are substantial in the geometrization of chemistry by making it possible to construct unique spatial mathematical images for both the SPECIES of simple chemical objects and those of SIMPLE CHEMICAL PROCESSES.

In order to construct the desired spatial mathematical model we formulate the following lemma:

**LEMMA 1.** Each species of atoms can uniquely be presented in the space  $V_M(3)$  by a corresponding point  $p$  with cylindrical coordinates  $(z,\rho,\varphi)$  in  $K_M$ , defined according to the rules:

- A1.  $z$  is equal to the number of neutrons in the nuclei of the presented species of atoms;
- A2.  $\rho$  is equal to the number of protons in the nuclei of the presented species of atoms;
- A3.  $\varphi = j(2\pi/8)$ , where  $j=1,\dots,8$ , is the number of the column (group) in the SFPS of atoms of the presented species.

The formulation of this lemma is the third step in proving the theorem (we constructed the desired approximate spatial mathematical model). Before to continue with the final step of the proof we shall comment the rules A1 and A3:

- The rule A1 represents the fact that the notion **SPECIES of atoms** is by definition (see [2, 3]) synonymous of the notion **isotope**, but not of the notion **chemical element**.
- The rule A3 represents the fact that only in the simplified tabular SFPS the notions **group of PS**, **column of single symbols** and **column of cells** are really synonymous. So, the mathematical modeling of the **columns** of filled cells in the simplified tabular SFPS is in fact a mathematical modeling of the basic chemical notions **group of PS** and **maximal stoichiometric valence** (or **characteristic valence**).

### INVESTIGATION OF THE APPROXIMATE MODEL; FIVE CHEMICAL AXIOMS

The following appropriate symbols were introduced:

- Capital **S** denotes **the set of all species of atoms** (i.e. all isotopes) **of chemical elements**, while small **s** denotes the **particular elements of S** (i.e. particular isotopes);
- **T<sub>0</sub>** denotes **the simplified tabular SFPS**;
- **f** denotes **the map defined by the rules A1 - A3**;
- **P** denotes **the set of all points p** ( $p \in V_M(\mathbf{3})$ ) **obtained by f**;
- the **right subscript** denotes **the atomic number i of the chemical element comprising the species of atoms under consideration**. For example, the symbol **Z<sub>i</sub>** denotes the set of **z**-coordinates of the images into **V<sub>M</sub>(3)** of all species of atoms of the chemical element with atomic number **i**;
- the **right bracketed subscript** denotes **the number j of that column (group) of the simplified tabular SFPS which comprises the species of atoms under consideration**. For example, the symbol **P<sub>(j)</sub>** denotes the set of images into **V<sub>M</sub>(3)** of all the species of atoms (isotopes) of the chemical elements of group **j** (i.e. the **j**-th column of the simplified SFPS).

Lemma 1 consists of two parts. The first one states that there exists an INVERTIBLE mapping **f** of the set **S** onto the set **P** of points **p**, where  $p = (s, T_0)$  and  $p \in V_M(\mathbf{3})$ :

$$(1) \exists f, \text{ such that } f(S, T_0) = P$$

The second part defines the invertible mapping **f** by the system of rules A1, A2 and A3 to obtain the coordinates (**z, ρ, φ**) of the points **p** ( $p \in P$ ) with respect to **K<sub>M</sub>** in **V<sub>M</sub>(3)**.

Using **f** and the other introduced symbols one can translate a number of basic chemical statements in mathematical language<sup>2</sup>.

**AXIOM 1.** Each chemical element has several different isotopes and their atoms differ from each other only in the number of neutrons.

In the language of set theory axiom 1 is reformulated as:

**AXIOM 1f.** For each **i** ( $i = 1, 2, \dots, 105, \dots$ ) the corresponding set **Z<sub>i</sub>** contains more than one element. In other words, the cardinal number of each **Z<sub>i</sub>** is always greater than 1.

This axiom points out that the set **S** of different species of atoms (not the set of different chemical elements) plays the role of a basic object in modern chemistry. That is why the contemporary forms of PS should be rather three-dimensional than two-dimensional.

**AXIOM 2.** The different species of atoms (i.e. different isotopes) of the same chemical element are of equal number of protons.

Let  $\{\rho\}_i$  be the set of **ρ**-coordinates of the mathematical images of all species of atoms of the chemical

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<sup>2</sup> The versions of statements translated in mathematical language are indicated by the symbol **f** at the end of the title.

element with atomic number  $\mathbf{i}$ .

**AXIOM 2f.** For each  $\mathbf{i}$  ( $\mathbf{i}=1,2,\dots,105,\dots$ ) the corresponding set  $\{\rho\}_{\mathbf{i}}$  contains a single element. In other words, the cardinal number of each set  $\{\rho\}_{\mathbf{i}}$  is equal to 1.

The single element of each set  $\{\rho\}_{\mathbf{i}}$  will be denoted hereafter by  $\rho_{\mathbf{i}}$ .

**AXIOM 3.** The number of protons in the nuclei of different atomic species of one and the same chemical element is equal to its atomic number  $\mathbf{i}$  in the periodic system.

**AXIOM 3f.** The  $\rho$ -coordinate of each mathematical image is equal to the atomic number  $\mathbf{i}$  of the element comprising the corresponding atomic species (presented by this mathematical image), i.e.:

$$(2) \quad \rho_{\mathbf{i}} = \mathbf{i}, \text{ for each } \mathbf{i} (\mathbf{i}=1,2,\dots,105,\dots)$$

**AXIOM 4.** For each atom the number of protons is equal to the total number of electrons (i.e. the atoms are electrically neutral chemical objects).

Using the last three axioms one expands the previously introduced **qualitative meaning** of the  $\rho$ -coordinates:

the  $\rho$ -coordinate of each of the constructed images represents simultaneously the following three characteristics of the corresponding species of atoms: the number of protons in the nuclei of atoms of this species; the total number of electrons in each atom of this species; the atomic number  $\mathbf{i}$  of this species of atoms (i.e. the atomic number  $\mathbf{i}$  of the chemical element comprising the atoms of this species).

As it follows from Lemma 1, the notions “group of PS” and “column of simplified tabular SFPS” are synonymous. From here one arrives at:

**AXIOM 5.** The maximal stoichiometric valence (characteristic valence) of all the species of atoms of chemical elements, belonging to one and the same column  $\mathbf{j}$  of the simplified tabular SFPS, is the same and equal to the column number  $\mathbf{j}$ .

From this axiom it becomes clear that:

the  $\varphi$ -coordinate of each image represents (with an accuracy of  $(2\pi/8)$ ) the following characteristics of the corresponding species of atoms: the number  $\mathbf{j}$  of the group (i.e. the number  $\mathbf{j}$  of the column in the simplified SFPS) comprising the atoms of this species, and their maximal stoichiometric valence.

The last axiom can also be translated into the language of set theory. Let  $\{\varphi\}_{(\mathbf{j})}$  denote the set of  $\varphi$ -coordinates of the images of all the species of atoms and all the chemical elements of the group of number  $\mathbf{j}$ . Then, according to rule A3, it follows:

**AXIOM 5f.** For each  $\mathbf{j}$  ( $\mathbf{j}=1,\dots,8$ ) the corresponding set  $\{\varphi\}_{(\mathbf{j})}$  contains only a single element. In other words, the cardinal number of the sets  $\{\varphi\}_{(\mathbf{j})}$  is equal to 1.

The single element of each set  $\{\varphi\}_{(\mathbf{j})}$  will be denoted hereafter by  $\varphi_{(\mathbf{j})}$ .

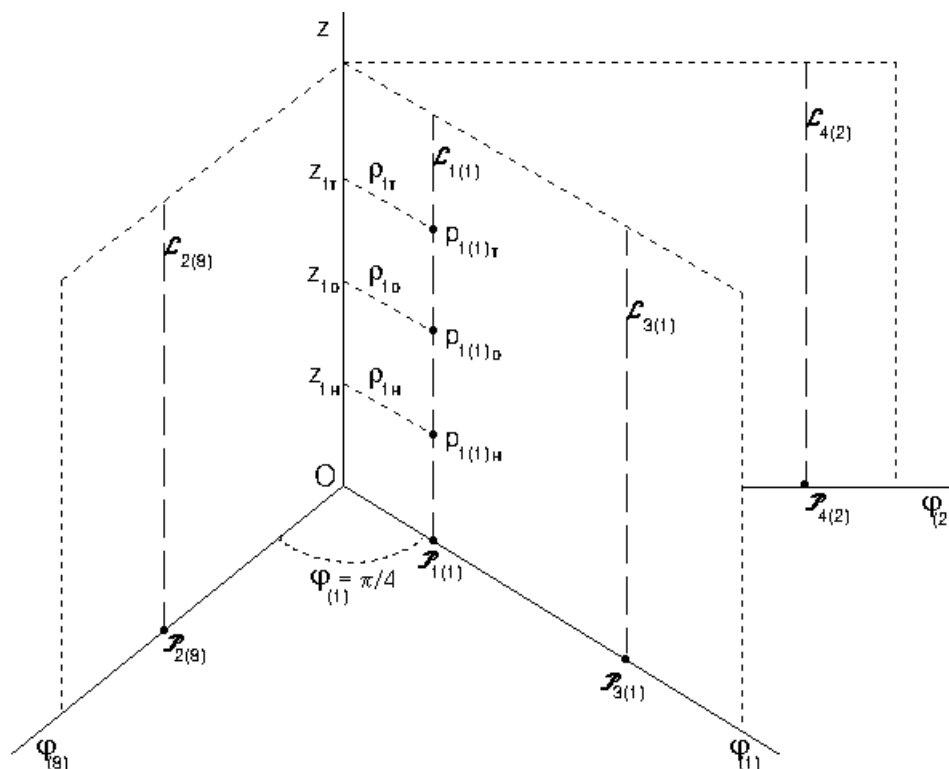
Using axioms 1f, 2f and 5f one obtains from Lemma 1 that:

The images in  $\mathbf{V}_M(\mathbf{3})$  of the different species of atoms (i.e., of different isotopes) of each chemical element have equal  $r$ -coordinates, equal  $j$ -coordinates and different  $z$ -coordinates, i.e.:

$$(3) \quad \mathbf{P}_{\mathbf{i}(\mathbf{j})} = (\mathbf{Z}_{\mathbf{i}}, \rho_{\mathbf{i}}, \varphi_{(\mathbf{j})})$$

The symbol of equality in (3) means that each element of the set  $\mathbf{Z}_{\mathbf{i}}$  is the first coordinate of the corresponding element of the set  $\mathbf{P}_{\mathbf{i}(\mathbf{j})}$ . For example,  $\mathbf{P}_{\mathbf{1}(\mathbf{1})}$  is the set of images of all the species of atoms of the chemical element with atomic number  $\mathbf{i}=1$  (see Fig. 2). The corresponding set  $\mathbf{Z}_{\mathbf{1}}$  contains three elements ( $\mathbf{z}_{\mathbf{1H}}=1$ ;

$z_{1D}=2; z_{1T}=3$ ) because the chemical element of order number  $i=1$  has three isotopes - hydrogen (**H**), deuterium (**D**) and tritium (**T**). Consequently, the set  $\mathbf{P}_{1(1)}$  contains three points  $\{\mathbf{p}_{1(1)H}; \mathbf{p}_{1(1)D}; \mathbf{p}_{1(1)T}\}$  of equal  $\rho$ -coordinates ( $\rho_1=1$ ), equal  $\varphi$ -coordinates ( $\varphi_{(1)}=(2\pi/8)$ ), and different  $z$ -coordinates ( $z_{1H} \neq z_{1D} \neq z_{1T}$ ).



**FIGURE 2. Mathematical representation of different species of atoms (isotopes) of the chemical element with atomic number  $i=1$ .** Shown are: Mendeleev's coordinate system  $\mathbf{K}_M=(O;z,\rho,\varphi)$ ; part of the semiplanes  $zO\varphi_{(1)}$ ,  $zO\varphi_{(2)}$  and  $zO\varphi_{(3)}$ ; the points  $\mathbf{p}_{1(1)H}$ ,  $\mathbf{p}_{1(1)D}$  and  $\mathbf{p}_{1(1)T}$  (i.e. the three-dimensional images of the atoms of hydrogen (**H**), deuterium (**D**) and tritium (**T**)); the points  $\mathcal{P}_{1(1)}$ ,  $\mathcal{P}_{2(8)}$ ,  $\mathcal{P}_{3(1)}$ , and  $\mathcal{P}_{4(2)}$  (i.e. the two-dimensional images in the plane  $z=0$  of the elements with atomic numbers  $i=1,2,3,4$ ); some of the lines  $\mathcal{L}_{1(1)}$ ,  $\mathcal{L}_{2(8)}$ ,  $\mathcal{L}_{3(1)}$  and  $\mathcal{L}_{4(2)}$ .

### PROOF OF THE UNIQUENESS AND THE VERACITY OF THE APPROXIMATE MODEL

In order to complete the proof of Theorem 1 we should perform the final, fourth, step of the conceptual scheme. In other words, we should check whether the point set  $\mathbf{P}=\mathbf{f}(\mathbf{S},\mathbf{T}_0)$  is simultaneously a **unique** mathematical model of different species of atoms and a **veracious** mathematical model of the initial simplified tabular SFPS. This statement would be correct if the constructed mathematical model satisfies the following conditions:

1. The correspondence  $\mathbf{f}$  between the set  $\mathbf{S}$  of all the species of atoms and the set  $\mathbf{P}$  of their mathematical images in  $\mathbf{V}_M(\mathbf{3})$  should be one-to-one correspondence (i.e. a bijection);
2. The image of each chemical element should contain only images of all the species of atoms (all the isotopes) of this chemical element;
3. The image of each group (i.e. of each column of the simplified tabular SFPS) should include only the images of all the chemical elements of this group;
4. The order of the chemical elements in each group should be analogous to that of their images in the image of this group;

5. The image of each period of the simplified SFPS should contain only the images of elements of this period;
6. The order of chemical elements in each period of the simplified SFPS and the order of their images in the image of this period should be analogous;
7. The order of groups and periods in the simplified SFPS and the order of their images should be analogous.

The verification of these conditions is given below.

1. The first condition is fulfilled because rules (A1, A2, A3) define an **invertible mapping  $f$**  of the set **S** onto the set **P** (see the definition of **invertible map**).

We proved so far the **uniqueness** of the model. In order to prove completely Lemma 1 we should show that the points of set **P** represent the SPECIES of the corresponding atoms. Thus, according to rules (A1, A2, A3) and Axiom 4, the coordinates of each of the constructed images represent (with respect to  $\mathbf{K}_M$ ) uniquely the physical composition of the corresponding atoms. On the other hand, the definition of the physical composition of a given simple chemical object determines uniquely its SPECIES [2, 3]. Therefore, the constructed images (the points  $\mathbf{p} \in \mathbf{P}$ ) represent in a unique way the SPECIES of the corresponding atoms, i.e. the set **P** is really a **unique** spatial mathematical model of the set **S** of different species of atoms. This completes the proof of Lemma 1.

2. Before verifying whether the constructed mathematical model obeys the **second** condition, let us see what are the images of the chemical elements. According to Axiom 1, the atoms of different isotopes of each chemical elements differ from each other only by the number of neutrons. As a result, according to rules A1, A2, A3, their images in  $\mathbf{V}_M(\mathbf{3})$  differ from each other only in their z-coordinates. On the other hand, according to rule A2 and Axiom 3f, the images of the isotopes of different chemical elements have different  $\rho$ -coordinates (as  $\rho_{\mathbf{i}} = \mathbf{i}$ ). Therefore, the following statement holds for each chemical element (i.e. for each  $\mathbf{i}$ ):

**COROLLARY 6.** The images of all species of atoms of the chemical element with atomic number  $\mathbf{i}$  (i.e. all points of the corresponding set  $\mathbf{P}_{\mathbf{i}(j)} = (\mathbf{Z}_{\mathbf{i}}, \rho_{\mathbf{i}}, \varphi_{(j)})$ ) lie on one and the same straight line  $\mathcal{L}_{\mathbf{i}(j)}$  which is parallel to the axis Oz and intersects the plane  $\mathbf{z} = \mathbf{0}$  in the corresponding point  $\mathcal{P}_{\mathbf{i}(j)} = (\rho_{\mathbf{i}}, \varphi_{(j)})$ .

Therefore, the second condition will be fulfilled if:

**DEFINITION 1.** The three-dimensional image of the chemical element with atomic number  $\mathbf{i}$  is a set of points  $\mathbf{P}_{\mathbf{i}(j)} = (\mathbf{Z}_{\mathbf{i}}, \rho_{\mathbf{i}}, \varphi_{(j)})$ , lying on the corresponding straight line  $\mathcal{L}_{\mathbf{i}(j)}$  which is parallel to the axis Oz and intersects the plane  $\mathbf{z} = \mathbf{0}$  of Mendeleev's  $\mathbf{V}_M(\mathbf{3})$  space in the point  $\mathcal{P}_{\mathbf{i}(j)} = (\rho_{\mathbf{i}}, \varphi_{(j)})$ .

This definition and Corollary 6 are illustrated in Fig. 2 for the chemical element with atomic number  $\mathbf{i} = \mathbf{1}$ .

3. The **third** condition is correct only for electrically neutral simple chemical objects (atoms). Before the verification of this condition, it is necessary to determine the images of different **groups (columns)** of the simplified SFPS. According to rule A3, the third condition is fulfilled, if:

**DEFINITION 2.** The image of the group (column) with a group number  $\mathbf{j}$  of the simplified SFPS is the corresponding set  $\mathbf{P}_{(j)}$  of points  $\mathbf{p}$ , lying on the semiplane  $\mathbf{zO}\varphi_{(j)}$  defined by the axis Oz and the ray  $\mathbf{O}\varphi_{(j)}$  (i.e. the set  $\mathbf{P}_{(j)}$  of the images with  $\varphi$ -coordinates equal to  $\varphi_{(j)}$ ). (See Figs. 2, 4a and 4b).

It follows from this definition that for electrically neutral simple chemical objects (i.e. for atoms):

- a) the images of all chemical elements of the group with a group number  $\mathbf{j}$  belong to the same semiplane  $\mathbf{zO}\varphi_{(j)}$ , defined by the axis Oz and the ray  $\mathbf{O}\varphi_{(j)}$  because their  $\varphi$ -coordinates are equal to  $\varphi_{(j)}$ ;
- b) only the images of chemical elements of the group with a group number  $\mathbf{j}$  belong to the semiplane  $\mathbf{zO}\varphi_{(j)}$  because only their  $\varphi$ -coordinates are equal to  $\varphi_{(j)}$ . For example, in addition to the straight line  $\mathcal{L}_{\mathbf{1}(1)}$ , straight lines  $\mathcal{L}_{\mathbf{3}(1)}$ ,  $\mathcal{L}_{\mathbf{11}(1)}$ ,  $\mathcal{L}_{\mathbf{19}(1)}$ ,  $\mathcal{L}_{\mathbf{37}(1)}$ ,  $\mathcal{L}_{\mathbf{55}(1)}$  and  $\mathcal{L}_{\mathbf{87}(1)}$ , of the family  $\mathcal{L}_{\mathbf{i}(1)}$  belong to the semiplane



$zO\varphi_{(1)}$  (see Fig. 2). In turn, the points  $\mathbf{p}$  of the corresponding sets  $\mathbf{P}_{1(1)}, \mathbf{P}_{3(1)}, \mathbf{P}_{11(1)}, \mathbf{P}_{19(1)}, \mathbf{P}_{37(1)}, \mathbf{P}_{55(1)}$  and  $\mathbf{P}_{87(1)}$  (i.e. the images of the different isotopes of all chemical elements of the first group ( $j=1$ ;  $i=1, 3, 11, 19, 37, 55, 87$ )) and only these images belong to the corresponding straight lines of the family  $\mathcal{L}_{i(1)}$ .

Thus, in the case of atoms the third condition is fulfilled. For the verification of the remaining conditions one needs two-dimensional instead of three-dimensional images of the chemical elements and groups (columns) of the simplified tabular SFPS.

It is known that the chemical properties of the different species of atoms are independent of the exact number of neutrons in their nuclei. Therefore, we can simply ignore the number of neutrons in entirely chemical considerations. This means that, in mathematical terms, instead of the constructed system of three-dimensional images of the different atomic species (i.e. instead of the points  $\mathbf{P}_{i(j)}=(Z_i, \rho_i, \varphi_{(j)})$ ) one can simply use their two-dimensional orthogonal projections in the plane  $z=0$  of  $\mathbf{V}_M(3)$  (i.e. the points  $\mathcal{P}_{i(j)}=(\rho_i, \varphi_{(j)})$ ). The coordinates  $(\rho_i, \varphi_{(j)})$  of these orthogonal projections are defined with respect to the polar Mendeleev's coordinate system  $\mathbf{K}_M=O\rho\varphi$ . In turn, this coordinate system is an orthogonal projection of the three-dimensional Mendeleev's coordinate system  $\mathbf{K}_M=(O; z, \rho, \varphi)$  onto the plane  $z=0$  of  $\mathbf{V}_M(3)$ .

For atoms it is easy to verify that:

**DEFINITION 3.** Each of the points  $\mathcal{P}_{i(j)}=(\rho_i, \varphi_{(j)})$  is a two-dimensional image of the corresponding chemical element (with atomic number  $i$ ) in the plane  $z=0$  of  $\mathbf{V}_M(3)$ .

Actually, the orthogonal projections of the three-dimensional images of the different species of atoms of the same chemical element in the plane  $z=0$  coincide. This geometrical result is important because it reveals the **qualitative meaning** of the points  $\mathcal{P}_{i(j)}$ :

**COROLLARY 7.** Each of the two-dimensional images of chemical elements (i.e. each of the points  $\mathcal{P}_{i(j)}=(\rho_i, \varphi_{(j)})$ ) represents the corresponding set of all atomic species (which are both different and chemically similar) in the plane  $z=0$  of  $\mathbf{V}_M(3)$ . Here, chemically similar atoms mean species of atoms with the same sets of chemical properties.

Let us see now what are the two-dimensional images of different **groups** (i.e. **columns**) of the simplified tabular SFPS. For atoms:

**DEFINITION 4.** The two-dimensional image of the group (column) with a group number  $j$  in the simplified SFPS is the set  $\mathcal{P}_{(j)}$  of those points  $\mathcal{P}_{i(j)}$ , that lie on the corresponding ray  $O\varphi_{(j)}$  in the plane  $z=0$  of  $\mathbf{V}_M(3)$ .

Furthermore, the two-dimensional image of each group of the simplified SFPS (i.e. the corresponding set  $\mathcal{P}_{(j)}$  of points  $\mathcal{P}_{i(j)}$ ) contains the two-dimensional images of all chemical elements of this group (i.e. all points  $\mathcal{P}_{i(j)}$ ) and only them. Therefore, the third condition is fulfilled (in the case of atoms) for both the three- and the two-dimensional images of the chemical elements and groups of the simplified SFPS.

For example, from the last two definitions it follows that the two-dimensional image of the third group ( $j=3$ ) of the simplified SFPS (i.e. the set  $\mathcal{P}_{(3)}$ ) contains all points  $\mathcal{P}_{i(3)}=(\rho_i, \varphi_{(3)})$  lying on the ray  $O\varphi_{(3)}$  where, in accordance with rule A3,  $\varphi_{(3)}=3\pi/4$ . In turn, the points  $\mathcal{P}_{i(3)}=(\rho_i, \varphi_{(3)})$ , according to Definition 3, are two-dimensional images in the plane  $z=0$  of lanthanides ( $i=57, \dots, 71$ ), actinides ( $i=89, \dots, 103$ ) and the other elements of the third group ( $i=5, 13, 21, 31, 39, 49, 81$ ).

**4. The fourth** condition is fulfilled if the order of the images of chemical elements in the image of the corresponding group of PS is along the direction of increasing atomic numbers  $i$ .

The following property of the sets  $\mathcal{P}_{\mathbf{i}(j)}$ , obtained by Eq. (2), is used to verify this condition in the **two-dimensional case**:

**PROPERTY 1.1.** The  $\rho$ -coordinate of each point  $\mathcal{P}_{\mathbf{i}(j)}$  is equal to its atomic number  $\mathbf{i}$ .

Using this property it is easy to prove that:

**COROLLARY 8.1.** For each  $j$  ( $j=1, \dots, 8$ ) the points  $\mathcal{P}_{\mathbf{i}(j)}$  of the corresponding set  $\mathcal{P}_{(j)}$  are ordered onto the corresponding ray  $O\varphi_{(j)}$  so that their sequential number  $\mathbf{i}$  increases with increasing  $\rho$ .

Corollary 8.1 is actually a proof that the constructed mathematical model of different species of atoms obeys the fourth condition in the two-dimensional case.

It is easy to verify that the three-dimensional images of chemical elements also obey the fourth condition. According to Corollary 6, all points of the corresponding set  $\mathbf{P}_{\mathbf{i}(j)} = (\mathbf{Z}_{\mathbf{i}}, \rho_{\mathbf{i}}, \varphi_{(j)})$  (which is a three-dimensional image of the chemical element with atomic number  $\mathbf{i}$ ) belong to the same straight line, denoted by  $\mathcal{L}_{\mathbf{i}(j)}$ . From the definition of the straight lines  $\mathcal{L}_{\mathbf{i}(j)}$  some properties of these follow:

**PROPERTY 1.** The sequential number  $\mathbf{i}$  of each of the straight lines  $\mathcal{L}_{\mathbf{i}(j)}$  is equal to the atomic number  $\mathbf{i}$  of the chemical element, whose three-dimensional image is the set of points  $\mathbf{P}_{\mathbf{i}(j)}$  defining this straight line.

**PROPERTY 2.** The number  $j$  of each of the straight lines  $\mathcal{L}_{\mathbf{i}(j)}$  is equal to the number  $j$  of the group of the simplified tabular SFPS containing the chemical element, whose three-dimensional image is the corresponding set of points  $\mathbf{P}_{\mathbf{i}(j)}$ .

**PROPERTY 3.** Each of the straight lines  $\mathcal{L}_{\mathbf{i}(j)}$  is parallel to the axis  $Oz$  and is situated in a distance  $\rho_{\mathbf{i}}$  from the axis  $Oz$ .

**PROPERTY 4.** The distance  $\rho_{\mathbf{i}}$  between the axis  $Oz$  and the corresponding straight line  $\mathcal{L}_{\mathbf{i}(j)}$  is equal to the sequential number  $\mathbf{i}$  of this line (because, according to Eq. (2),  $\rho_{\mathbf{i}} = \mathbf{i}$ ).

**PROPERTY 5.** All straight lines  $\mathcal{L}_{\mathbf{i}(j)}$  are parallel to each other (because they are parallel to  $Oz$ ).

**PROPERTY 6.** All straight lines  $\mathcal{L}_{\mathbf{i}(j)}$  are perpendicular to the plane  $z=0$  (because they are parallel to the axis  $Oz$  which is perpendicular to the plane  $z=0$ ).

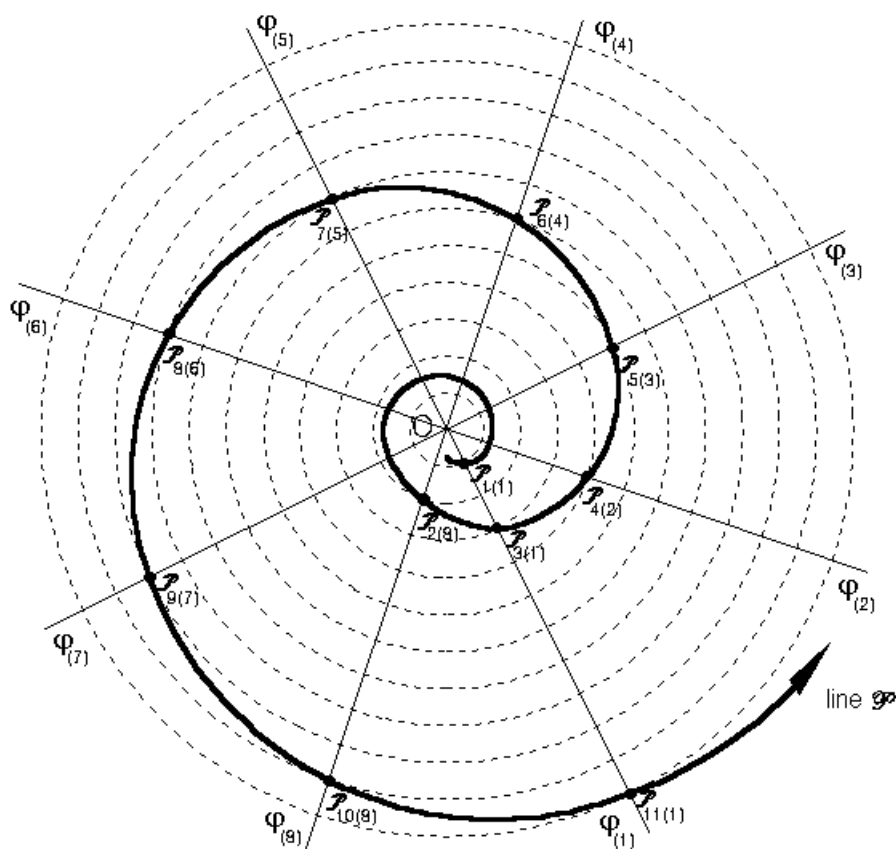
**PROPERTY 7.** Each of the straight lines  $\mathcal{L}_{\mathbf{i}(j)}$  belongs to the corresponding semiplane  $zO\varphi_{(j)}$ , and only those lines with a number  $j$  equal to the number  $j$  of this semiplane belong to it (this property follows from the definitions of the lines  $\mathcal{L}_{\mathbf{i}(j)}$  and from Definition 2). (For example, all lines  $\mathcal{L}_{\mathbf{i}(3)}$  ( $\mathbf{i}=5, 13, 21, 31, 39, 49, 57, \dots, 71, 81, 89, \dots, 103$ ) and only the lines of this family belong to the semiplane  $zO\varphi_{(3)}$ .)

With the aid of these properties it is easy to prove that the fourth condition is fulfilled for the three-dimensional case. According to Property 7, all straight lines  $\mathcal{L}_{\mathbf{i}(j)}$  with the same number  $j$ , and only they, belong to the corresponding semiplane  $zO\varphi_{(j)}$  (having the same number  $j$ ). In addition, according to Properties 3 and 5, the lines  $\mathcal{L}_{\mathbf{i}(j)}$  lying on the same semiplane  $zO\varphi_{(j)}$ , are parallel to each other, as well as to the axis  $Oz$ . In accordance with Property 4 one can formulate:

**COROLLARY 8.** For each  $j$  the straight lines  $\mathcal{L}_{\mathbf{i}(j)}$  lying on the corresponding semiplane  $zO\varphi_{(j)}$ , are ordered so that their number  $\mathbf{i}$  increases with increasing  $\rho$ .

Therefore, if we accept that the distance between the axis  $Oz$  and the image of the corresponding chemical element is also a criterion for the order in the three-dimensional case<sup>3</sup>, then, (according to Def. 1 and 2, and Corollary 8) the fourth condition is fulfilled for the three-dimensional case as well.

In order to verify the fulfilling of the last three conditions in the two-dimensional case, we construct one **auxiliary line**  $\mathcal{P}$  through the points  $\mathcal{P}_{\mathbf{i}(j)}$  in the plane  $z=0$  of  $\mathbf{V}_M(3)$  which: a) passes through each of the points  $\mathcal{P}_{\mathbf{i}(j)} = (\rho_{\mathbf{i}}, \varphi_{(j)})$  only once; b) in the regions corresponding to lanthanides ( $\mathbf{i}=57, \dots, 71$ ) and actinides



**FIGURE 3. Construction of the auxiliary line  $\mathcal{P}$  in the plane  $z=0$  of  $V_M(3)$ .** Shown is the starting part of the line  $\mathcal{P}$ , where it passes through the points  $\mathcal{P}_{\mathbf{i}(j)}$  of numbers  $\mathbf{i}=1,2,\dots,11$  and  $j=1,2,\dots,8$  (i.e. through the two-dimensional images of the first 11 chemical elements). The radii  $\rho_{\mathbf{i}}$  of the dashed circumferences are equal to  $1,2,\dots,11$ , respectively.

( $\mathbf{i}=89,\dots,103$ ), is a part of the ray  $O\varphi_{(3)}$ ; c) in the regions corresponding to elements of number  $\mathbf{i}=26,27,28$ ;  $44,45,46$ ;  $76,77,78$ , is a part of the ray  $O\varphi_{(8)}$  (these elements belong to the eighth group of the simplified SFPS); d) in the remaining regions looks like a spiral. In addition, let the line  $\mathcal{P}$  pass through the points  $\mathcal{P}_{\mathbf{i}(j)}$  so that: e) the sequence of numbers  $\mathbf{i}$  of the points  $\mathcal{P}_{\mathbf{i}(j)}$ , through which it subsequently passes, to be a monotonously increasing sequence. The starting part of the line  $\mathcal{P}$  is shown on Fig. 3.

The auxiliary line  $\mathcal{P}$  constructed in such a way allows to determine the two-dimensional images of different periods of the simplified SFPS. Actually, if we consider the subsequent turns of the line  $\mathcal{P}$ , confined in the range  $0 < \varphi_{(j)} \leq 2\pi$ , then (with the aid of rule A3 and Definition 3) we obtain the following two statements:

**COROLLARY 9.** The two-dimensional images of all chemical elements which belong to the same short period of the simplified SFPS (i.e. the corresponding points  $\mathcal{P}_{\mathbf{i}(j)}$ ), belong to the corresponding single turn of the line  $\mathcal{P}$ .

**COROLLARY 10.** The two-dimensional images of all chemical elements, belonging to one and the same long period of the simplified SFPS (i.e. the corresponding points  $\mathcal{P}_{\mathbf{i}(j)}$ ), belong to the corresponding double turn of the line  $\mathcal{P}$  (i.e. to the corresponding two consecutive single turns of  $\mathcal{P}$ ).

On the basis of these corollaries the following definition can be formulated:

**DEFINITION 5.** The two-dimensional image of each period of the simplified SFPS in the plane  $z=0$  is the

<sup>3</sup> The criterion of order is just the same in the two-dimensional case (according to Corollary 9).

set of the corresponding points  $\mathcal{P}_{\mathbf{i}(j)}$  (i.e. the set of the two-dimensional images of the chemical elements belonging to this period), which: (1) belong to the corresponding single turn of the auxiliary line  $\mathcal{P}$  if the period is short; (2) belong to the corresponding double turn of  $\mathcal{P}$  if the period is long.

**5. The fifth condition** is fulfilled in the two-dimensional case because, according to Corollaries 9 and 10, the two-dimensional images of all chemical elements of a given period lie on the corresponding single or double turn of the line  $\mathcal{P}$ . At the same time, only the two-dimensional images of the chemical elements of the corresponding period lie on this single or double turn.

**6. The sixth condition** is also fulfilled in the two-dimensional case. Let us choose as positive direction the direction of movement down the line  $\mathcal{P}$  of a monotonous increase in the numbers  $\mathbf{i}$  of points  $\mathcal{P}_{\mathbf{i}(j)}$ . On the other hand, for all tabular forms of PS the following statement holds: the chemical elements in each period are ordered in a sequence which corresponds to the increase of their successive atomic numbers  $\mathbf{i}$ .

**7.** Before verifying the fulfillment of **the seventh condition**, it is necessary to note that the following is valid for the simplified tabular SFPS:

- the groups are ordered in a rightward sequence according to the increase in their numbers  $\mathbf{j}$ ;
- the periods are ordered in a downward sequence according to the increase in their numbers.

These two statements are also valid for the two-dimensional images of the groups and periods of the simplified tabular SFPS. From Definition 4 and rule A3, it follows that:

- the two-dimensional images of the groups of the simplified tabular SFPS are ordered in a sequence corresponding to the increase in their group numbers  $\mathbf{j}$  (if the movement is counterclockwise).

In addition, if we number the subsequent turns of the line  $\mathcal{P}$  by the numbers of the corresponding periods of the simplified SFPS it follows from Definition 5 that:

- the two-dimensional images of the periods of the simplified SFPS are ordered in a sequence corresponding to the increase of their numbers (if the movement is in the positive direction of the line  $\mathcal{P}$ ).

Finally, the two-dimensional image of each chemical element (i.e. the corresponding point  $\mathcal{P}_{\mathbf{i}(j)}$ ) is a point of intersection of the two-dimensional images of the group and the period to which this element belongs.

Thus, we have shown that the seventh condition is fulfilled in the two-dimensional case. **This completes the proof of Theorem 1 for the two-dimensional case.** The final steps of proving this theorem for the three-dimensional case follows below.

To check the fulfillment of the last three conditions in the three-dimensional case, we construct an **auxiliary surface**  $\mathcal{B}$  in  $\mathbf{V}_M(3)$ , which obeys the following conditions (see Fig. 4a): a) it is perpendicular to the plane  $z=0$ ; b) it intersects the plane  $z=0$  in the auxiliary line  $\mathcal{P}$ .

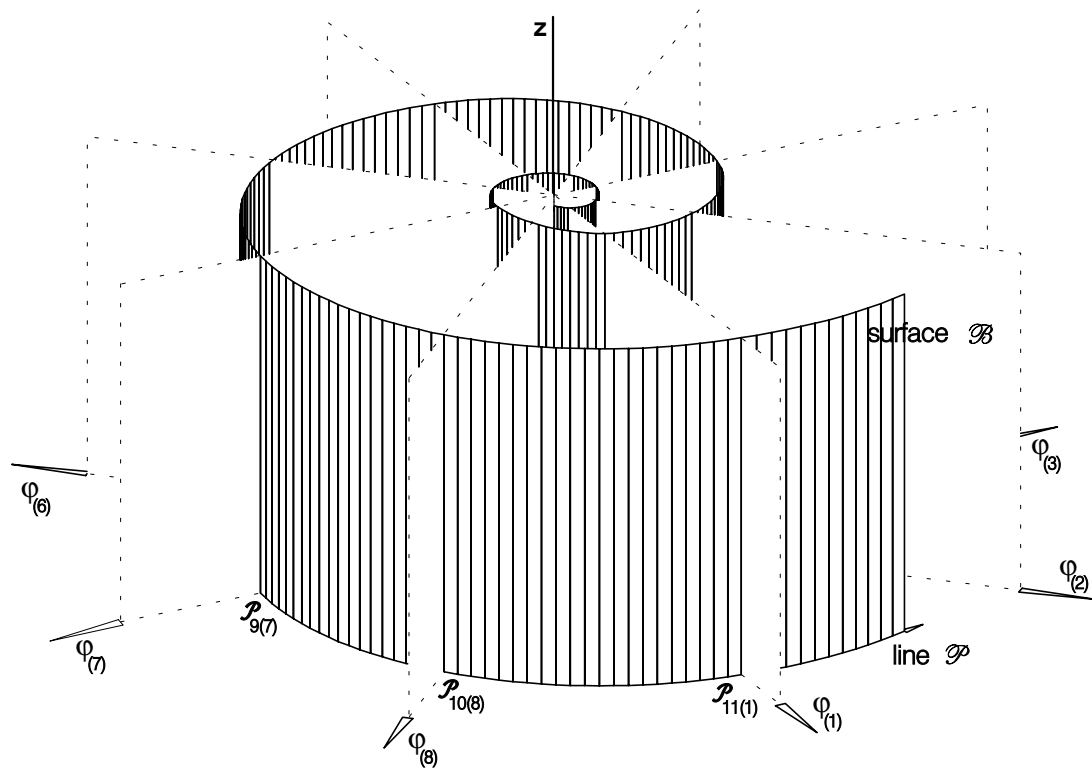
From the aforesaid conditions and definitions of  $\mathcal{B}$ ,  $\mathcal{P}$  and  $\mathcal{L}_{\mathbf{i}(j)}$ , it is evident that (see Fig. 4b):

**COROLLARY 11.** All straight lines of the family  $\mathcal{L}_{\mathbf{i}(j)}$  ( $\mathbf{i}=1, \dots, 105, \dots$ ;  $\mathbf{j}=1, \dots, 8$ ) lie in the surface  $\mathcal{B}$ .

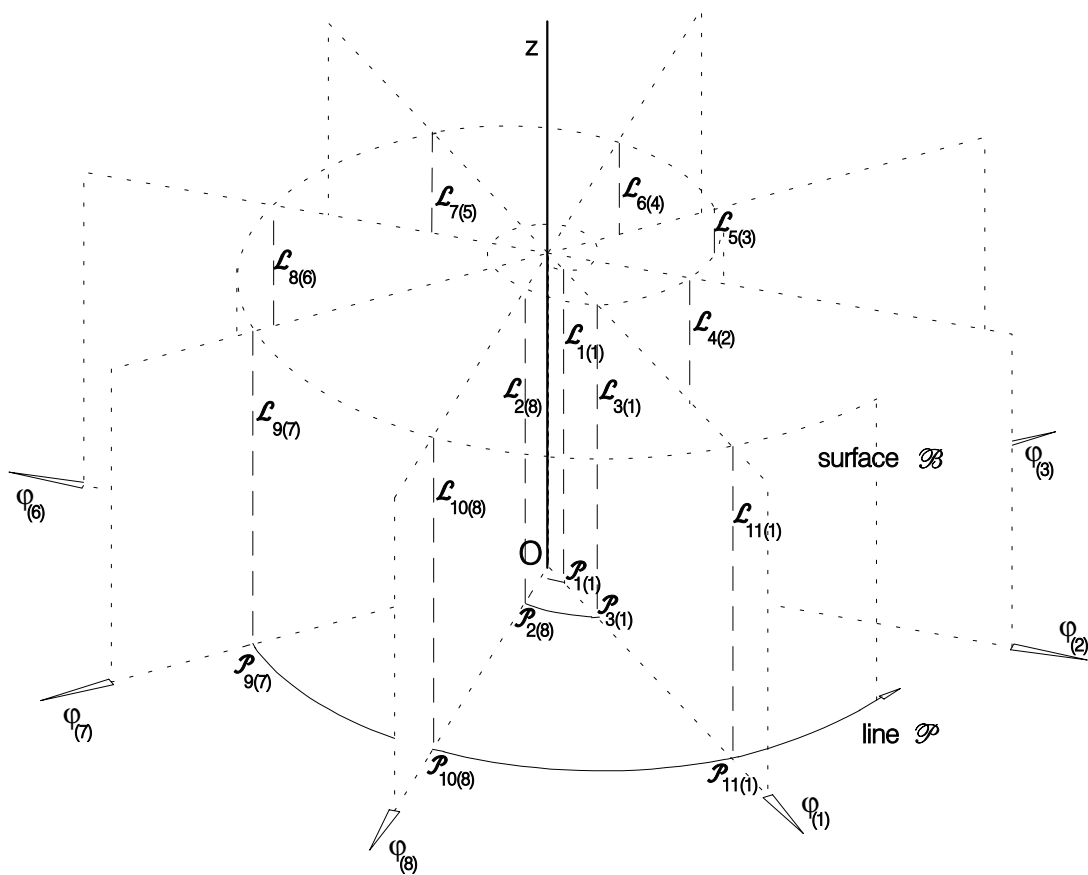
The surface  $\mathcal{B}$  allows to determine the three-dimensional images of different periods of the simplified tabular SFPS. Let us consider the subsequent turns of the surface  $\mathcal{B}$ , confined in the range  $0 < \varphi_{(j)} \leq 2\pi$ . Then, using the definitions of  $\mathcal{B}$ ,  $\mathcal{P}$ ,  $\mathcal{L}_{\mathbf{i}(j)}$  and Corollary 11, we obtain:

**COROLLARY 12.** In each turn of the surface  $\mathcal{B}$  lie only those straight lines of the family  $\mathcal{L}_{\mathbf{i}(j)}$ , whose points of intersection  $\mathcal{P}_{\mathbf{i}(j)}$  with the line  $\mathcal{P}$  lie in the corresponding turn of this line.

As an illustration of Corollaries 11 and 12, Fig. 4b shows the part of  $\mathcal{B}$ , where the lines  $\mathcal{L}_{\mathbf{i}(j)}$  with numbers  $\mathbf{i}$  from 1 to 11 lie (i.e., the lines containing the three-dimensional images  $\mathbf{P}_{\mathbf{i}(j)}$  of the chemical elements with atomic numbers  $\mathbf{i}=1, 2, \dots, 11$ ).



**FIGURE 4a. Construction of the auxiliary surface  $\mathcal{B}$  in  $V_M(3)$ .**



**FIGURE 4b. Auxiliary surface  $\mathcal{B}$  in  $V_M(3)$ . Shown is the starting part of the surface  $\mathcal{B}$ , where it passes through the lines  $\mathcal{L}_i(j)$  of numbers  $i=1,2,\dots,11$  and  $j=1,2,\dots,8$ .**

From the last two corollaries and Definition 1 it follows:

**COROLLARY 13.** The three-dimensional images of all chemical elements, which belong to one and the same short period of the simplified SFPS (i.e., the points of the corresponding sets  $\mathbf{P}_{\mathbf{i}(\mathbf{j})}$ ), lie on the corresponding single turn of the surface  $\mathcal{B}$ .

**COROLLARY 14.** The three-dimensional images of all chemical elements, which belong to the same long period of the simplified SFPS (i.e. the points of the corresponding sets  $\mathbf{P}_{\mathbf{i}(\mathbf{j})}$ ), lie on the corresponding double turn of the surface  $\mathcal{B}$  (i.e., on the corresponding subsequent single turns of  $\mathcal{B}$ ).

The last two corollaries and Definition 1 enable us to formulate the following definition:

**DEFINITION 6.** The three-dimensional image of each period of the simplified SFPS is a set of points  $\mathbf{p}$  (obtained by the map  $\mathbf{f}$ ), which: 1) lie on the corresponding single turn of the surface  $\mathcal{B}$  if the period under consideration is short; 2) lie on the corresponding double turn of  $\mathcal{B}$ , if the considered period is long.

**5.** To show that the **fifth condition** is also fulfilled in the three-dimensional case, we use the fact that, according to Corollaries 13 and 14, the three-dimensional images of all chemical elements of a given period lie on the corresponding single or double turn of the auxiliary surface  $\mathcal{B}$ . At the same time, (according to Corollary 12 and Definition 1) only the three-dimensional images of the chemical elements of the period under consideration lie on this single or double turn. From these geometric reasons it follows that the fifth condition is fulfilled in the three-dimensional case.

**6.** To verify **the sixth condition** in the three-dimensional case, we use the following **criterion**: the order of the three-dimensional images of chemical elements (i.e. of the sets  $\mathbf{P}_{\mathbf{i}(\mathbf{j})}$ ) is the order of lines  $\mathcal{L}_{\mathbf{i}(\mathbf{j})}$  in the three-dimensional image of each period. This **three-dimensional criterion of order** is a **generalization** of the two-dimensional one, because: 1) according to Definition 1, the points of each set  $\mathbf{P}_{\mathbf{i}(\mathbf{j})}$  define uniquely the corresponding line  $\mathcal{L}_{\mathbf{i}(\mathbf{j})}$  (i.e. the line of the same numbers  $\mathbf{i}$  and  $\mathbf{j}$ ); 2) the two-dimensional criterion of order is obtained there from as a limit when going from three- to two-dimensional images of the chemical elements.

From this generalized criterion of order it follows that the three-dimensional images of chemical elements (the sets  $\mathbf{P}_{\mathbf{i}(\mathbf{j})}$ ) obey the sixth condition because: (1) each point  $\mathcal{P}_{\mathbf{i}(\mathbf{j})}$  is an orthogonal projection in the plane  $\mathbf{z}=\mathbf{0}$  of the corresponding line  $\mathcal{L}_{\mathbf{i}(\mathbf{j})}$  (where the point set  $\mathbf{P}_{\mathbf{i}(\mathbf{j})}$  lies) and each turn of the line  $\mathcal{P}$  is an orthogonal projection on the corresponding turn of the surface  $\mathcal{B}$  on the same plane; (2) the points  $\mathcal{P}_{\mathbf{i}(\mathbf{j})}$  obey the sixth condition; (3) the order of lines  $\mathcal{L}_{\mathbf{i}(\mathbf{j})}$  in each turn of the surface  $\mathcal{B}$  is **isomorphic** to the order of their orthogonal projections (the points  $\mathcal{P}_{\mathbf{i}(\mathbf{j})}$ ) on the orthogonal projection of this turn of the surface  $\mathcal{B}$  (i.e. on the corresponding turn of the line  $\mathcal{P}$ ).

**7.** The fulfillment of **the seventh condition** in the three-dimensional case follows directly from the definitions of the three-dimensional images of the chemical elements, groups and periods. The role of a criterion of order in this case is played by **the mutual disposition** of the lines  $\mathcal{L}_{\mathbf{i}(\mathbf{j})}$ , semiplanes  $\mathbf{zO}\varphi_{(\mathbf{j})}$  and turns of the surface  $\mathcal{B}$ .

Therefore, we showed that the seven conditions are fulfilled in the three-dimensional case as well. Consequently, the point set  $\mathbf{P}=\mathbf{f}(\mathbf{S},\mathbf{T}_0)$  is a unique spatial mathematical model of the set  $\mathbf{S}$  of different species of atoms and a veracious mathematical model of the starting simplified tabular SFPS. **This completes entirely the proof of Theorem 1.**

## DISCUSSION AND CONCLUSIONS

In the foregoing we founded, constructed and analyzed an approximate spatial mathematical model of the set **S** of different species of atoms and Mendeleev's law of periodicity, presented by the simplified tabular SFPS **T<sub>0</sub>**.

The first conclusion from the approximate model is:

**1.** There are no models in chemistry analogous to the constructed three-dimensional mathematical model of the simplified tabular SFPS.

This state of the art is due to the following two facts:

a) All current representations of PS (independently of their particular form) are based on different **chemical GENERA** (i.e. on the notion chemical element), but not on different **chemical SPECIES**<sup>4</sup> (i.e. on the notion **species of atoms** or **isotopes**). So, **the current representations of PS describe the transformation of the sets of the chemical properties of simple electrically neutral chemical SPECIES when their chemical GENUS changes.**

b) On the other hand, the constructed approximate mathematical model is based on the notion of simple electrically neutral **chemical SPECIES** (i.e. on the **species of atoms** or **isotopes**), but not on the notion of **chemical GENUS** (i.e. on the chemical element). That is why **this model describes the transformation of the general set of properties of the different atomic species when their SPECIES AFFILIATION** (not their **GENERIC AFFILIATION**<sup>5</sup>) **changes.**

The difference between the constructed three-dimensional image of PS and its tabular representations becomes clear when one takes into account the following fact:

When the **generic affiliation** of a given simple chemical object is transformed, its **species** affiliation is transformed as well. However, the reverse statement is generally not true because there are a number of processes transforming the affiliation of **species** of simple chemical objects, but not their **generic** affiliation. All processes described by the classical chemistry belong to this group. For example, such processes are all processes of ionization and recombination of simple chemical objects, all processes of transformation of one isotope of a chemical element into another, etc.

This analysis leads to the following conclusions:

**2.** The constructed three-dimensional mathematical model is of a better resolving power than the current representations of the periodic system (independently of their particular form), because it represents uniquely different chemical SPECIES (i.e. different atomic species) rather than different chemical genera (i.e. chemical elements).

**3.** The resolving power of the constructed two-dimensional mathematical model (in the plane **z=0** of **V<sub>M</sub>(3)**) is equal to that of the current representations of PS.

**4.** The following axiom can be formulated:

**AXIOM 6.** The numbers which define uniquely the actual physical composition of each simple chemical object (i.e., the number of neutrons, protons and electrons in each atom or elementary ion) are always positive integers.

From this axiom it follows that the SPECIES of each simple chemical object is defined uniquely by the

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<sup>4</sup> See footnote 1 on page 2.

<sup>5</sup> There are a number of processes (such as  $\alpha$ -decay,  $\beta$ -decay, radioactive decay, thermonuclear synthesis, etc.), transforming the **genus** of simple chemical objects.

corresponding triad of positive integers presenting the numbers of neutrons, protons and electrons in this simple chemical object. It is clear from this statement that only the corresponding discrete sets of points which belong to the Mendeleev's space  $V_M(\mathbf{3})$  have a chemical meaning in all the constructed mathematical models of different species of simple chemical objects and the Mendeleev's law of periodicity<sup>6</sup>. This conclusion suggests that chemistry is most probably a particular manifestation of the abstract **nonarchimedean** geometry whose axiomatics has been studied by D. Hilbert [5].

By founding, constructing and analyzing the approximate spatial mathematical model we **axiomatize** (i.e. formulate a logical structure of definitions, axioms, theorem, lemma, corollaries and properties) this part of the language of chemistry, which introduces the set **S** of different species of **atoms**. The axiomatic formulations of the same part of the language of chemistry, based on two more accurate mathematical models **Q** and **G** [3], are analogous [2].

By founding, constructing and analyzing the two generalized spatial mathematical models **Q** and **G** [3] the axiomatization of the first part of the language of chemistry (the statics of simple chemical objects) is entirely completed [2] and the axiomatization of the second part of the language of chemistry (dynamics of the simple chemical objects) is grounded.

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<sup>6</sup> Just this peculiarity of the geometric representation of the basic chemical notions and relations makes it necessary to introduce different auxiliary geometrical objects in the corresponding mathematical model. For example, the role of such objects in the approximate model is played by the lines  $\mathcal{L}_i(j)$ ,  $\mathcal{P}$  and the surface  $\mathcal{B}$ .