A Study On The Determination Of The Dynamic displacement Magnification Factor Which Varies With The Parameter Of Frequency Caused By The Anelastic Behaviour Of Material

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Abstract

This paper gives a method to determine the dynamic displacement magnification factor which is according to C.Zener's theory of anelasticity [1] and some microscopic mechanism of anelastic phenomenon(internal friction, elastic creep, stress relaxation and variation of dynamic modulus with frequency of measurements or temperature) which are contrary to the theory of elasticity but are shown by real solids. In the discussion of physical interpretation of anelasticity presented in this paper we are concerned primarily with the physical origin of the anelastic effects. All such effects are different manifestation of the lack of uniqueness of the relation between stress and strain. C.Zener derived the relation between internal friction(tan   and stress function f(t) from the assumption that the true relation between stress and strain are linear and apply the Boltzmann's supperposition principle to the mechanical model of a standard linear solid. From the relation between internal friction and logarithmic decrement which is the indirect method of measuring internal friction in the case of small tan   - the natural logarithm of the ratio of successive amplitudes- the author derived a method to calculate the disspated coefficient B and the anelastic damping factor γ. By use of B and γ this paper gives a formula to determine the dynamic displacement magnification factor β which can be used for structural components in one degree of freedom forced fibrant system. The proposed method is found to be simple and its result is rather close to the test data.

Keywords: anelastic,damping factor, dynamic displacement magnification factor, internal friction

The Anelastic Behaviour Of Material

C.Zener described the anelastic property of solid in his book"Elasticity And Anelasticity Of Metals". He also studied very definite questions regarding the kinetics of changes in the micro-structure. That metals does not perfectly obey the theory of elasticity even at low stress levels. They manifest deviations from perfect elastic behavior. Some irreversible processes had happened. It dissipates a part of energy which gains. We have observed the gradual decay in amplitude of vibration during free oscillation and stop at last. Under forced vibration the amplitude has to reach a maximum definite quantity when resonance and don't enlarge to infinity. [3] The effect of amplitude of vibration which quickly decay with time in any structure components is mainly caused by internal friction rather than external damping.

The classical theory of elasticity is essentially the development of all the consequences arising from the application of Hook's law to elementary regions. According to the classical theory of elasticity, stress and strain are uniquely related. The relation between applied force and deformation are also linear provided that it is varied so slowly that no vibrations result(quasi-static forces). But there are other effects which are contrary to the theory of elasticity. They have been shown by real solids. That are internal friction, elastic creep, stress relaxation and variation of dynamic modulus with frequency of measurements. All such effects are different manifestation of the lack of uniqueness of the relation between stress and strain. In the pre-plastic range, the property of solids in virtue of which stress and strain are not single-valued function of one another in that low stress range in which no permanent set occurs and in which the relation of stress to strain is still linear is defined as "Anelasticity".

Classical theory of elasticity is a macro-theory which neglects the effects of microstructure in real solids. So it can't explain the effects as stated above and has the limitation of application. The value of the classical theory lies not in its precise describtion of the behavior of solids under applied forces but in its description of this behavior with sufficient accuracy for most practical purposes.

Let B be the dissipated coefficient[10] which denotes the degree of energy dissipation of material.

$$B = \Delta E / E$$

Where E is the total energy of certain system. ΔE is the dissipated energy. Under small amplitude condition, ΔE and E are both in direct proportion to the square of the amplitude of vibration.

$$A_1^2 = E \qquad 1$$

$$A_2^2 = E - \Delta E = E(1-B) \qquad 2$$

We already know that the logarithmic decrement is defined as the natural logarithm of the ratio of successive amplitudes. So we have

$$\ln(A_1^2 / A_2^2) = \ln(1/ 1-B) \qquad 3$$

$$\Lambda = \ln(A_1 / A_2) = 1/2 \ln(1/ 1-B) \qquad \text{(log. dec.)} \qquad 4$$

Eq. 4 denotes that the logarithmic decrement for amplitude is just one half that for energy. From Eq. 1,2 we get (when it far less than 1)

$$\Delta E / E = \frac{A_1^2 - A_2^2}{A_1^2} \qquad 2\frac{(A_1 - A_2)}{A_2} \qquad 2 \ln(A_1 / A_2) \qquad 2 \Lambda$$ [12]

We also have B 2Λ, when B is small compared to unity in Eq. 4. The anelastic damping factor in the case of small tan is defined as

$$\gamma = \tan \quad = \Lambda / \pi = \frac{\ln(1/1-B)}{2\pi} \quad [1] \qquad 5$$

γ is in direct proportion to B . It is evident that the above relation must become invalid for large values of the internal friction because the internal damping of specimen itself will overdamp the vibration resulting in aperiodic motion.

The Calculation Of Dissipated Coefficient B And Internal Friction γ

C.Zener has derived the relation between internal friction and stress function as follows[1]:

$$\gamma = \tan \quad = -\frac{\pi}{2}\left(\frac{d \ln f}{d \ln t}\right)_{t=T/8} \qquad 6$$

According to the connotation of Eq. 6 we convert the form of $\left(\frac{d \ln f}{d \ln t}\right)_{t=T/8}$ to suit the corresponding form in this paper.

$$\gamma = \tan \quad = -\frac{\pi}{2}\,\frac{\log(1-B)}{\log(T/8)} \qquad 7$$

Let Eq. 5 equal to Eq. 7 and make logarithm variation, we have

$$\log\left[\frac{\ln(1/1-B)}{2\pi}\right] = -\frac{\pi}{2}\,\frac{\log(1-B) + 10}{\log(T/8) + 10}$$

$$T = 8X\,10^{\left\{-\frac{\log(1-B) + 10}{\log\left[\frac{\ln(1/1-B)}{2\pi}\right]\frac{2}{\pi}} - 10\right\}} \qquad 8$$

Where T is the period of vibration. Eq. 8 is the basic equation for calculating the dssipated coefficient B. Several B values are listed in Table 1 with T varies from 4.09E-07s to 375s.

For different materials, we use the conversion coefficient R_x to obtain corresponding dissipated coefficient B_x which is equal to $R_x B$.

$$B_x = \left(\frac{E\, C_x\, \rho_x}{E_x\, C\, \rho} \right) B = R_x B \qquad 9$$

Where E_x, C_x, ρ_x and E, C, ρ represent the modulus of elasticity, specific heat and density of requisite material and structural steel respectively. The dissipated coefficient B_x of different materials are dependent on their physical attribution which are mainly dependent on E, C and ρ . Several R_x values of structural materials are listed in Table 2. Utilize Eq. 8,9 from which we can calculate the internal friction γ by use of Eq. 5.

Dynamic Response On One Degree Of Freedom Forced Vibrating System

Now we consider the differential equation of motion of simply supported beam under periodic load. A simply supported beam under periodic load is shown in Fig. 1. Where ω is angular velocity and α is angular displacement.

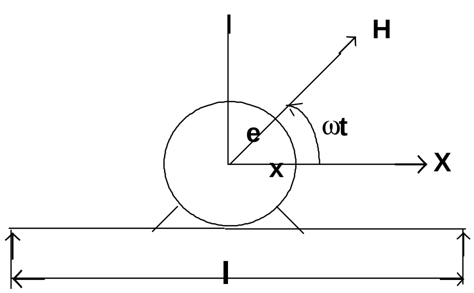

Fig. 1

$$\omega = \frac{d\,\alpha}{d\,t} \quad \text{rad. / s}$$

$$X = e \cos \omega t$$

$$\dot{X} = - e\, \omega \sin \omega t$$

The centrifugal force

$$H = \frac{mV^2}{e} = \frac{m\, e^2\, \omega^2 \sin^2 \omega t}{e} = m\, e\, \omega^2 \sin^2 \omega t$$

$$H_{max} = m\, e\, \omega^2$$

The periodic force apply to the beam is

$$P = H \sin \omega t$$

In the consideration of internal friction(γ) of material, we can write the equation of motion as follows:

$$F + Q + R = H \sin \omega t$$

$$m \ddot{y} + Q + \gamma Q = H \sin \omega t$$

$$m \ddot{y} + (1 + \gamma) Q = H \sin \omega t$$

$$\ddot{y} + (1 + \gamma) \frac{C}{m} y = \frac{H}{m} \sin \omega t$$

$$\ddot{y} + k^2 y = \frac{H}{m} \sin \omega t \qquad 10$$

Where $F = m \ddot{y}$ is inertial force, $Q = C y$ is restoring force and $R = \gamma Q$ is anelastic damping force.

$$k = (1 + \gamma) \frac{C}{m} = 1 + \gamma \quad \phi = 1 + \gamma \quad 2 \pi f$$

Where ϕ is natural circular frequency.

$$f = \frac{\phi}{2 \pi}$$

The homogeneous solution of Eq. 10 is

$$y_1 = C_1 \cos kt + C_2 \sin kt$$

Let particular solution

$$y_2 = A \sin \omega t$$

$$\dot{y}_2 = \omega A \cos \omega t$$

$$\ddot{y}_2 = - \omega^2 A \sin \omega t$$

Substitute it in Eq. 10 , we have

$$A = \frac{H}{m} \frac{1}{k^2 - \omega^2}$$

That is

$$y_2 = \frac{H}{m} \frac{1}{k^2 - \omega^2} \sin \omega t$$

The general solution of Eq. 10 is

$$y = y_1 + y_2 = C_1 \cos kt + C_2 \sin kt + \frac{H}{m} \frac{1}{k^2 - \omega^2} \sin \omega t \qquad 11$$

Now , we pay attention to the third term of Eq.11 . That is the amplitude caused by forced vibration.

$$A = \frac{H}{m} \frac{1}{k^2 - \omega^2} \sin \omega t = \frac{H}{m} \frac{1}{(1+\gamma)\, 4\pi^2 f^2 - \omega^2} \sin \omega t$$

$$= \frac{H}{m\, 4\pi^2 f^2} \frac{1}{\left[(1+\gamma) - \frac{4\pi^2 \omega_1^2}{4\pi^2 f^2}\right]} \sin \omega t$$

$$= \frac{H}{m \frac{C}{m}} \frac{1}{\left[(1+\gamma) - \frac{\omega_1^2}{f^2}\right]} \sin \omega t$$

$$= \frac{H}{C} \frac{1}{\left[(1+\gamma) - \frac{\omega_1^2}{f^2}\right]} \sin \omega t$$

$$= \frac{H y}{Q} \frac{1}{\left[(1+\gamma) - \frac{\omega_1^2}{f^2}\right]} \sin \omega t$$

$$= \frac{H y}{H \sin \omega t} \frac{1}{\left[(1+\gamma) - \frac{\omega_1^2}{f^2}\right]} \sin \omega t$$

$$= \frac{1}{\left[(1+\gamma) - \frac{\omega_1^2}{f^2}\right]} y = \beta y \qquad 12$$

$$\beta = \frac{1}{\left[(1+\gamma) - \frac{\omega_1^2}{f^2}\right]} \qquad 13$$

Where β is the dynamic displacement magnification factor which varies with the parameter of frequency and internal friction. y is the static deflection under load

P. f and ω_1 are natural and forced vibration frequency respectively. When

$$f = \omega_1$$

$$\beta = \frac{1}{\gamma} \qquad 14$$

The dynamic factor β reaches to a maximum limit value and will not enlarge to infinity.

Minoru Wakabayashi gives the dynamic displacement magnification factor under steady forced vibration state [8] as follows:

$$D_d = \left| \frac{V}{V_{st}} \right| = \left[(1- \beta 1^{\wedge 2})^{\wedge 2} + 4 \xi^{\wedge 2} \beta 1^{\wedge 2} \right]^{-1/2} \qquad A$$

When

$$\beta 1 = \frac{\omega_1}{f} = 1$$

$$D_d = \frac{1}{2\xi}$$

Compare to Eq. 14 , that is

$$\gamma = 2\xi \qquad 15$$

When $\beta 1 = 1$, in Fig. 2-5, reference 8, pp. 40, the values of D_d are the same as in Eq. 14. While $\gamma = 2\xi = 0$, Eq. 13 is as same as Eq. A[9]. But the value ξ in Eq. A can not be calculated theoretically. The term damping force F_d in equation of motion is the multiplication of damping coefficient C and displacement velocity. That is the well known V.Thomson and Voight hypothesis.

Now we determine the constant C_1 and C_2 in Eq. 11. When

t = 0, y(t) = 0, then $C_1 = 0$

t = 0, $\dot{y}(t) = 0$, then

$$- C_1 k\sin kt + C_2 k\cos kt + \frac{H}{m} \frac{1}{k^{\wedge 2}- \omega^{\wedge 2}} \omega \cos \omega t = 0$$

$$C_2 = - \frac{H}{m} \frac{1}{k^{\wedge 2}- \omega^{\wedge 2}} \frac{\omega}{k}$$

Eq. 11 becomes

$$y = -\frac{H}{m}\,\frac{1}{k^2-\omega^2}\,\frac{\omega}{k}\sin kt + \frac{H}{m}\,\frac{1}{k^2-\omega^2}\sin \omega t$$

$$= \beta y \sin \omega t - \beta y \frac{\omega}{k}\sin kt = \beta y\left(\sin \omega t - \frac{\omega}{k}\sin kt\right) \qquad 16$$

y is maximum when $\sin kt = 0$, $\sin \omega t = 1$ therefore

$$y_{max} = \beta y$$

The values of dynamic displacement magnification factor β which varies with γ and ω_1^2 / f^2 are listed in TABLE 3. TABLE 3 denotes that :

If we can control the frequency ratio ω_1^2 / f^2 0.2 in the range of anelastic damping factor γ = 0.01-0.2, then we can control the magnification factor β within 1-1.235.

When γ is constant, we can control β 1.5 while $1+ \gamma - (\omega_1^2 / f^2)$ 0.7.

o increase the vibrant isolation coefficient, it is necessary to decrease the natural frequency . We may choose the isolator which leads to ω_1^2 / f^2 1. In practise, the dampers stiffness is , as a rule , chosen so that ω_1^2 / f^2 = 2.5-5 [4] at moderate damping.

TABLE 1

B	T	f(1/T)
1.000000000E-05	4.094560000E-07	2.442263450E+06
5.000000000E-05	9.628530000E-07	1.038580330E+06
1.000000000E-04	1.502530000E-06	6.655423400E+05
5.000000000E-04	5.433220000E-06	1.840529600E+05
1.000000000E-03	1.093460000E-05	9.145295000E+04
5.000000000E-03	9.378710000E-05	1.066245000E+04
1.000000000E-02	3.301050000E-04	3.029340000E+03
2.000000000E-02	1.581140000E-03	6.324600000E+02
3.000000000E-02	4.797210000E-03	2.084600000E+02
4.000000000E-02	1.179900000E-02	8.475000000E+01
5.000000000E-02	2.563740000E-02	3.901000000E+01
6.000000000E-02	5.128170000E-02	1.950000000E+01
7.000000000E-02	9.665260000E-02	1.035000000E+01
8.000000000E-02	1.741620000E-01	5.740000000E+00
9.000000000E-02	3.029920000E-01	3.300000000E+00
1.000000000E-01	5.124630000E-01	1.950000000E+00
1.100000000E-01	8.469760000E-01	1.180000000E+00

1.133956585E-01	1.000000000E+00	1.000000000E+00
1.200000000E-01	1.373250000E+00	7.282000000E-01
1.300000000E-01	2.190920000E+00	4.560000000E-01
1.400000000E-01	3.447950000E+00	2.900000000E-01
1.500000000E-01	5.363130000E+00	1.870000000E-01
1.600000000E-01	8.258820000E+00	1.210000000E-01
1.700000000E-01	1.260848000E+01	7.900000000E-02
1.800000000E-01	1.910590000E+01	5.200000000E-02
1.900000000E-01	2.876582000E+01	3.500000000E-02
2.000000000E-01	4.307032000E+01	2.300000000E-02
2.553376950E-01	3.754121196E+02	2.660000000E-03

TABLE 2

Structural Material	E (Kg/cm2)	C	ρ (Kg/cm3)	R_x
Steel	2.10E+06	0.107	0.00785	1
Concrete	1.55E+05-3.65E+05	0.2	0.0025	8.065-3.525
Brick	3.00E+05	0.19	0.0018	2.8502
Timber	1.00E+05	0.3	0.0006	4.5003

TABLE 3

T	B	$\frac{\omega^2}{f^2}$ β γ	0	0.2	0.4	0.6	6.25	25
1.372931E-04	0.006263487	0.001	0.999	1.248	1.664	2.494	-0.191	-0.042
.054405152	0.060898632	0.01	0.99	1.235	1.639	2.439	-0.191	-0.042
4.375782909	0.145364	0.025	0.98	1.121	1.6	2.353	-0.191	-0.042
648.4030066	0.269597308	0.05	0.952	1.117	1.539	2.222	-0.192	-0.042
1522024.823	0.466511908	0.1	0.909	1.111	1.429	2	-0.194	-0.042
1.41136E+12	0.715390456	0.2	0.833	1	1.25	1.666	-0.198	-0.042
1.16891E+26	0.918997408	0.4	0.714	0.833	1	1.25	-0.206	-0.042
1.30433E+50	0.976945889	0.6	0.625	0.714	0.833	1	-0.215	-0.043

The Comparison Of Theoretical Calculation And Test Data

There are rare reports about the tests of magnification factor and the records of test data are also imperfect. We utilize the Merlin L. James's paper[2] which reports the test of dynamic properties of reinforced and prestressed concrete structure components. Sixteen concrete beams were subjected to sinusoidal exciting forces of varying magnitudes for the purpose of evaluating the flexural rigidity and internal damping properties. Use the magnification factor curves, Fig. 4 of reference 2, to compare with the results which calculated in this paper by use of Eq. 13. The theoretical calculated results are very close to the test data. These

results are listed in Table 4.

The exciting force and natural frequency are obtained in Table 3, reference 2. E_x is calculated by the following formula.

$$E_x = 4\,w\,l^3 f_1^2 \,/\, \pi^2 I g$$

Where

l = 116.5 in.
W = The weight of beam (Table 2, reference 2)
f_1 = Natural frequency (Table 3, reference 2)
$I = 4X8^3 / 12 = 170.6666\ in^4$
$g = 386.0885827\ in / s^2$

1 in^3 = 16.387064 cm^3
1 lb = 0.453599081 Kg
1 p.s.i. = 0.070307 Kg / cm^2
ρ_x = W / 4X 8 X 120 Kg / cm^3

Beam 21 through 26 were prestressed by pretensioning one 3/8 in. diameter steel cable to 14000 lb. The maximum tensile stress and compressive stress in the concrete of the prestressed beam was calculated as 210 and 1023 psi. respectively. Each of the prestressed beams had one or two easily visible tensile cracks across the upper face of the beam and extending varying amounts down into the cross section when they were dilivered.

TABLE 4 (1)

Beam No.	Exciting Force	f_1	E_x (Kg/cm2)	ρ_x (Kg/cm2)	C_x	R_X
#6	4	34	2.529348875E+05	2.306692853E-03	0.2	4.560126496
	5.7	33.8	2.499679350E+05	"	"	4.614252153
	7.5	32	2.240530491E+05	"	"	5.147955301
	8.5	31.5	2.171060918E+05	"	"	5.312679495
	10	31	2.102685354E+05	"	"	5.485438321
	15.5	30.5	2.035403798E+05	"	"	5.666762942
#5	1	39.1	3.533223731E+05	2.436444326E-03	0.2	3.448110772
#21	2	40.1	3.551339495E+05	2.328318099E-03	0.2	3.278279502
	3.1	40	3.533649664E+05	"	"	3.294691389
	6.3	39.8	3.498401013E+05	"	"	3.327887062
	12.5	39.4	3.428434761E+05	"	"	3.395801375
	25	39	3.359175236E+05	"	"	3.465816057
#22	3	40.2	3.580123761E+05	2.335526514E-03	0.2	3.261989944

TABLE 4 (2)

Beam No.	B	B_X	$\beta_{cal.}$	$\beta_{prc.}$	Note
#6	0.05190161	0.236678814	23.2645	22	
	0.051984425	0.239869245	22.9092	21	
	0.0527539	0.271574719	19.8289	19	
	0.0529764	0.281446634	19.0103	18.5	
	0.05320306	0.291842104	18.2075	18	
	0.053434005	0.302797839	17.4204	17	
#5	0.049966859	0.172291264	33.2279	32.5	
#21	0.049621592	0.162673448	35.39	34.8	Visible tensile
	0.04965567	0.163600108	35.1707	32	cracks
	0.04972412	0.165476256	34.7341	28	
	0.049862221	0.169322199	33.8692	26	
	0.05000195	0.173297561	33.0155	25	
#22	0.049587612	0.161754292	35.6101	35	

We also utilize reference 3, examples 6 and 18. The results are listed in Table 5, It shows that the theoretical calculated results are still agree with the empirical data.

TABLE 5 (1)

Example	f	ω_1	E_X (Kg/cm2)	ρ_X(Kg/cm3)	C_X	R_X
6	23.5	16.7	2.1000E+06	7.8500E-03	0.107	1
18	29.4	16.7	1.6500E+05	2.5000E-03	0.2	7.5762

TABLE 5 (2)

Example	B	B_X	β cal.	β prc.
6	0.062371122	0.062371122	1.9793	2
18	0.062371122	0.472536627	1.2835	1.46

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?100 !PROGRAM FOR CALCULATING THE DD MAGNIFICATION FACTOR (COE.B)
?110 INPUT PROMPT "T=?":T
?120 FOR B=1E-5 TO 1.0  STEP 1E-5
?130     LET A=LOG10(1-B)+10
?140     LET C=LOG(1/(1-B))
?150     LET D=LOG10(C/(2*PI))
?160     LET E=D*(2/PI)
?170     LET F=A/E
?180     LET G=-F-10
?190     LET T1=8*10^G
?200     PRINT "T1=";T1,"B=";B
?210     LET T2=T
?220     IF  T1-T2=>1E-9 THEN
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?230        EXIT FOR
?240     END IF
?250 NEXT B
?260
?270 INPUT PROMPT "B=?":B
?280 LET E1=2.1E+6                 !MODULUS OF ELASTICITY OF STEEL
?290 LET C1=0.107                  !SPECIFIC HEAT OF STEEL
?300 LET D1=0.00785                !DENSITY OF STEEL
?310 INPUT PROMPT "EX=?":EX        !M.E.OF USING MATERIAL
?320 INPUT PROMPT "CX=?":CX        !S.H.OF USING MATERIAL
?330 INPUT PROMPT "DX=?":DX        !DENSITY OF USING MATERIAL
?340 LET RX=(E1*CX*DX)/(EX*C1*D1)
?350 PRINT "RX=";RX
?360 LET BX=RX*B
?370 PRINT "BX=";BX
?380
?390 INPUT PROMPT "BX=?": BX
?400 LET H=LOG(1/(1-BX))
?410 LET GAMA=H/(2*PI)             !GAMA= INTERNAL FRICTION
?420 PRINT "GAMA=";GAMA
?430
 440 INPUT PROMPT "GAMA=?": GAMA
 450 INPUT PROMPT "J=?":J          !J=w^2/f^2 FREQUENCY RATIO
 460 LET BETA= 1/(1+GAMA-J)        !BETA=DYNAMIC FACTOR
 470 PRINT "BETA=";BETA
 480
 490 END
 500
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