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Mathematical harmony and information cyclicity

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Abstract: The paper discusses the relation between the recently discovered principle of information cyclicity and the mathematical conception of harmonious relation called golden section. The existence of the connection of the principle and the conception is proven by two ways, i.e. with geometrical approach and the statements of information theory. The connection enables to consider the golden section as one of information constants. The analysis of optimal information system consisting of two independent components has demonstrated the existence of information optimality for one, two and three-dimensional spaces. The main conclusion of the paper is that the fundamental mathematical conceptions of harmony and balance for all their abstractness are of information nature in terms of information theory

Keywords: entropy, quantity of information, golden section, optimality

1 Introduction

The approximate connection ($æ_o$* f_o* z_o = 1) exists between the optimum accuracy coefficient $æ_o$ = 1/2ð as the factor of information cyclicity and the golden section f_o = 0.618 through the base of decimal numeration z_o = 10 (proven as being optimal) [1]. The aim of this paper is the more profound study of the relation between golden section and information cyclicity, as well as how this relation influences the optimality concerning a coordinate space.

The study of the relation between golden section and information cyclicity is briefly carried out with the two different ways based on 1) geometrical approach using six-sided polygon and inscribed rectangle, and 2) information theory using such information characteristics as entropy (*H*) and quantity of information (*I*). In addition the brief theoretical analysis of information optimality in the framework of different coordinate systems is carried out with the mathematical model of two-component optimal system.

2 Geometrical connection between golden section and information cyclicity

The golden section (also called *golden mean*) is defined as: *a ratio between two portions of a line or the two dimensions of a plane figure, in which the lesser of the two is to the greater as the greater is the sum of both* [2].

The golden section corresponds to that rectangle with sides *A(n)* and *B(n)* as functions of the number (*n*) of a regular polygon, which is inscribed into the optimum regular polygon when a side of this polygon is equal to *A*(*n*). Under this condition the following equations are true:

A(n)/B(n) = *B(n)*/[*A(n)* + *B(n)*] = tan (   ) = f_o = 0.618,

where: *A(n) = d sin* (   );
B(n) = d cos (   );
= ð   ð(*n*   2)/2*n*   ð/2 = ð/*n*;
d = the diameter of the circle, into which the rectangle and the polygon are inscribed.

From this equation the optimum polygon shall consist of the number n_o of sides that in the rounding off (Ö) expression is calculated as follows:

n_o = Ö {ð/arc tan (0.618)} = 6.

Thus, the direct connection between information cyclicity and golden section may be considered as proven.

This result meets the requirements of information cyclicity in its representation in the form of integer. The information cycle expressed as an integral value ç satisfies the condition of permissible information accuracy: |ç - 2ð |/2ð < 1/2ð. In accordance with this condition, ç can take a value 6 or 7 only.

The analogous result is achieved when using geometrical proof within the range of permissible by information redundancy optimum accuracy coefficient (from 1/4ð to 1/2ð) as follows:

n_o = arg [(1/2ð   ä(*n*)   1/4 ð],

where: ä(*n*) = 1 - *B(n)/d* = 1 – cos (ð/*n*) = the relative difference between the diameters of the circles circumscribing and of and inscribing in a regular polygon. The six and seven sided regular polygons only satisfy this condition.

The integers 6 and 7 are of great structural and classification significance in the nature [3].

3 Informational connections between golden section and information cyclicity

In terms of information theory [4] and according to the principle of information cyclicity [5] the optimal system of two components, representing a complete group of independent events, is characterized by weights (relative significances) as the estimates of probabilities $P_1 + P_2 = 1$ and the following condition of their optimal ratio:

1/4ð < $(P_2 / P_1)_o$ < 1/2ð

Under this condition the system is characterized by optimum redundancy of information. When P_2 / P_1 > 1/2ð, the system is informatively insufficient, and when P_2 / P_1 < 1/4ð, its redundancy is in excess of necessity.

Inasmuch as the information optimality occupies the range, and is not concentrated on a point, the question is to what extent the limiting entropies and quantities of information, i.e. their maximum and minimum values in respect to the optimum and full range of information, are in mathematical harmony (in terms of golden section), if any.

3.1 Limiting information characteristics

A complete system of two components involves some limiting information characteristics as is illustrated with the information scale (Figure 1).

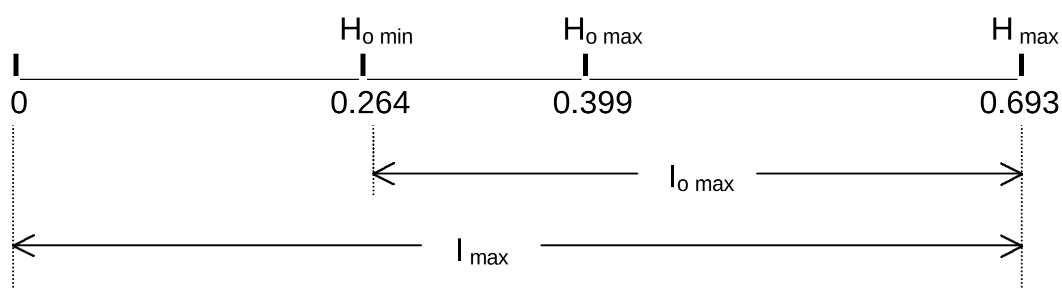

Figure 1. The information scale of the system of two components Involves: H_{max} = the maximum possible entropy; $H_{o\,max}$ = the maximum optimal entropy; $H_{o\,min}$ = the minimum optimal entropy; I_{max} = the maximum possible quantity of information; $I_{o\,max}$ = the maximum optimal quantity of information.

The limiting information characteristics were calculated with the following expressions:

$I_{max} = H_{max} = \ln 2$;
$I_{o\,max} = H_{max}$ $H_{o\,max}$;
$H_{o\,max} = - P_{11} \ln P_{11} – P_{21} \ln P_{21}$;
$H_{o\,max} = - P_{12} \ln P_{12} – P_{22} \ln P_{22}$,

where: P_{11} = 2ð /(1 + 2ð), P_{21} = 1/(1 + 2ð);
P_{21} = 4ð /(1 + 4ð), P_{22} = 1/(1 + 4ð).

3.2 Limiting information relations

The comparison of information cyclicity and golden section is of advantage in that it have bearing upon the same model of comparing objects, namely the ratios of values that can be presented as intercepts.

In order to compare the ratios of information characteristics with golden section, factor f_o is reasonable to consider within the range of permissible information accuracy: f_o (1+ 1/4ð) = 0.618 ± 0.049. Thus the satisfactory comparison matches this accuracy, and the estimation error should not exceed ± 7.9%. The results of calculation are given in the table below.

Information ratio	Result (R)	Estimation error $(1 - R/f_o)100\%$
$H_{o\ max}/H_{max}$	0.576	+ 6.8
$H_{o\ min}/H_{o\ max}$	0.662	- 7.1
$(H_{o\ min}/H_{o\ max})^{1/2}$	0.617	+ 0.16
$I_{o\ max}/I_{max}$	0.619	- 0.16

Clearly, in all cases the estimation error is less then 7.9% that conforms the information ratios as being mathematically harmonious.

4 Geometrical simulation of the optimal two-component information system

The simplest optimal information system consisting of two independent components, which probabilities represent the complete group of independent events, may geometrically be presented as follows:

$$a_{(m)}^{m} / (1 - a_{(m)}^{m}) = \text{æ}_{\text{ð}} = 1/2\text{ð},$$

or $$a_{(m)}^{m} = (2\text{ð} + 1)^{-1/m},$$

where $a_{(m)}^{m}$ and $(1 - a_{(m)}^{m})$ are parts of geometrical formation of *m* power of complexity (*m* = 1, 2, 3, . . .) that numerically equal to respective probabilities.

Clearly, according to the normalization the following condition is true for the rounding off (Ö) geometrical presentation of the system:

$$\text{Ö}\ \{(2\text{ð} + 1)^{-1/m}\}\ \ 0.5$$

This condition allows to claim that the information optimality exists only for one (*m* = 1), two (*m* = 2), and three (*m* = 3) – dimensional spaces.

Another noteworthy discovery consists in the fact that with estimation error less than 1/4ð (which is permissible according to the principle of information cyclicity) the following equations are true:

$$1 - a_{(1)}^{1} = 1 - \text{æ}_{\text{ð}};$$

$1 - a_{(2)}^{2} = f_o$;
$1 - a_{(3)}^{3} = 1 - {}^{o}$,

where: ^{o} = 0.5 – the mathematical balance number (here $a_{(3)}^{3} = 1 - a_{(3)}^{3}$).

4 Discussion and conclusions

4.1 The obtained results demonstrate the highly satisfactory coordination of information cyclicity and golden section both in terms of geometry and information theory. Evidently, the connection between $æ_o$ and f_o enables to consider the golden section as one of information constants.

4.2 The geometrical simulation of informatively optimal system of two components has enabled the following:

- to find the mathematical balance number ^{o} = 0.5 as one more information constant,
- to demonstrate that the sufficient information on the unit area (n = 2) and unit volume (n = 3) is attainable when the unit intercept (n = 1) is divided in two parts corresponded to the harmony and balance, and
- to claim the existence of information optimality only for one, two and three – dimensional spaces.

4.3 Apparently, the same informativeness, forming with different parts of the constituents of unit formation, is in full accord with Shannon's statement that the measurement is relative to the coordinate system, and if we change coordinates, the entropy will in general change [4].

4.4 The results, possibly, point out to the existence of deep objective connections between the information optimality and mathematical harmony in the nature.

4.5 There is the inevitable and still not resolved problem of describing measurement and other information in a qualitative sense since the first publication of and in addition to the pure statistical Shannon's theory, which has been discussed in many works and in various aspects [6, 7]. In the author's opinion, the conceptions of sufficiency, harmony and balance, interpreted here in terms of information theory can serve as one of instruments in establishing the correlation between logical and semantic information.

4.6 The recognition of information nature of the conceptions of optimality, harmony and balance, being a contribution to information theory, is of interdisciplinary concern in achieving the optimum information, particularly for system-defined purposes, e.g. in biology, psychophysics, metrology and other applied sciences, industrial sphere, etc.

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